Toward Global Food Security: Transforming OCP Through Analytics

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Abstract. Humanity relies on cultivated lands to feed itself and thrive. Fertilizers are responsible for 30%–50% of food production, and phosphate, the naturally occurring form of phosphorus, which does not have a substitute, is an essential component of fertilizers. OCP, based in Morocco, is the world’s largest phosphate mining and processing company and therefore plays a critical role in global food security. Over the past decade, OCP, in collaboration with Dynamic Ideas, an analytics consulting company, developed a mixed-integer optimization model to holistically optimize its entire sales and supply chain—from the mines to physical treatments, to chemical facilities, to inventory facilities, and to the port for global distribution. The optimization model brings clarity to a complex supply chain, informs management decisions throughout OCP, and has consistently resulted in an improvement of over 20% in earnings before interest, taxes, depreciation, and amortization (EBITDA) annually. This amounted to over $2.3 billion in the period 2015–2020 (23.6% of the cumulative EBITDA of $9.9 billion over this period). This incremental profitability has fueled OCP’s financing capacity; as a result, OCP is implementing a $20 billion capital expenditures (CAPEX) program. The first phase of the CAPEX program led to the doubling of OCP’s mining capacity and the tripling of its fertilizer production capacity. As a result, OCP increased its fertilizer production capacity by eight million tons in the past decade. The model enabled OCP to produce customized fertilizers that helped improve agricultural yields, which in turn led to increased food production, especially in Africa. The increased production of fertilizers and the availability of customized fertilizers have contributed toward global food security. By demonstrating the interconnectedness of all OCP businesses, the model contributed to creating a culture of collaboration, innovation, and entrepreneurship across the company while breaking the existing silos among departments. This transformation led to the establishment of an analytics-based problem-solving approach throughout OCP and a successful executive education class at the Massachusetts Institute of Technology, thus enriching the university.

Introduction

OCP Group (OCP S.A.) (formerly Office Cherifien des Phosphates) is a Moroccan-based phosphate rock miner, phosphoric acid manufacturer, and phosphate fertilizer producer and exporter, approximately 94% of which is owned by the Moroccan state. Founded in 1920, OCP has grown into the largest phosphate mining and processing company in the world and contributes 31% of the global phosphates trade. Phosphorus is an element that is key to global food security because, as phosphate rock, it is an essential component of fertilizers, and it does not have a substitute. OCP has access to more than 70% of the world’s phosphate rock reserves (United States Geological Survey 2020). It employs nearly 20,000 people in Morocco and in a number of international subsidiaries and has offices worldwide (e.g., in the United States, Brazil, India, China, Ethiopia, and Singapore). In 2018, its revenues amounted to $6 billion, which represents 5% of the gross domestic product of Morocco.
OCP extracts about 40 million tons of ore (raw rock) per year from more than 45 fronts at eight mining locations. Each mine can have several fronts. A front is a geographical sector where planning and operations can be managed autonomously, independent from other areas and/or fronts of the same mine. Each location can deliver up to 10 different grades of raw rock, which may undergo physical processing and blending before being either exported or further processed into 9 grades of phosphoric acid or 43 grades of fertilizers in more than 42 plants across multiple locations. Managing such a complex supply chain is challenging because of the large number of decision variables, including extraction planning, raw rock beneficiation (i.e., the process of improving the quality of the raw ore), rock blending, and chemical processing allocation. All these operational decisions must be aligned with the global sales pipeline.

Dynamic Ideas is an analytics consulting company cofounded by Professor Dimitris Bertsimas and Dr. Gina Mourtzinou. Several of its senior team members were trained in operations research at the Massachusetts Institute of Technology (MIT). Dynamic Ideas and OCP have collaborated since 2010 and have developed a multiperiod mixed-integer optimization model that spans all OCP’s operations, from mining to sales. This model brings together all the major operational decisions of the company on the same decision platform, enabling managers to collaborate to maximize overall corporate profit. It currently runs on an ad hoc basis, monthly, quarterly, annually, and over multiple years, depending on the nature of decisions to be made. By transforming the way managers understand the interconnectedness of the company, the platform has helped stimulate greater cooperation, transparency, and alignment on common objectives. It has contributed to breaking down the organizational silos and educating managers on the interdependency of their actions, and it has helped OCP to build a corporate culture of innovation and collaboration.

The model optimizes the entire OCP business process—from the mines to physical treatments, to the chemical facilities, to inventory facilities, and to the port for global distribution. It allows managers to make all decisions in conjunction with each other and, in addition, allows users to investigate how changes in one part of the supply chain affect all the others. Based on years of research and development, the model runs in less than a minute and enables many teams within OCP to investigate what-if scenarios and obtain insights regarding bottlenecks and binding constraints. It automatically creates reports that are used as production guidelines in every mine, every physical treatment, and every chemical facility. Moreover, the model provides managers with a better understanding of OCP’s supply chain and especially its bottlenecks. Through better integrated pricing and business steering, the platform helped define an optimal product portfolio and production strategy that maximizes OCP’s margin on variable costs, and more important, allows OCP to lead the effort for food security in Africa and other locations worldwide.

The Role of OCP in Global Food Security

Humanity relies on cultivated lands to feed itself and thrive. All plants need three essential nutrients to grow: nitrogen, phosphorus, and potassium, each of which is absorbed from the soil. Therefore, fertilizers, the providers of the key nutrients that sustain soil fertility and enable the production of crops that can meet humanity’s growing food needs, play a crucial role in feeding the world’s population. Fertilizers are responsible for 30%–50% of food production (Stewart et al. 2005), and these percentages are expected to increase in the future. The world population is forecasted to exceed nine billion by 2050, and dietary habits are increasingly protein rich, especially in developing countries such as India and China (whose populations are consuming increasingly more poultry and meat). Therefore, crop production is projected to increase by more than 60% between 2005–2007 and 2050 (Food and Agriculture Organization of the United Nations 2012). Because the total farmed land area is expected to increase by less than 5% by 2050 (Food and Agriculture Organization of the United Nations 2012), farming intensification must deliver the bulk of the increases in food production. Balanced and responsible fertilization has a critical role to play in future food security.

Achieving global food security is a daunting challenge, especially in developing regions. Africa is one of the regions that is most affected by hunger. In 2020, 20.3% of the population of sub-Saharan Africa (SSA) was undernourished, the highest rate in the world (World Bank 2021). Grain yields have been lagging significantly in SSA, directly impacting food availability in the region. The region is using an average of 21 kilograms per hectare of fertilizers, far from the 110 kg/ha global average for developing countries (Food and Agriculture Organization of the United Nations 1995). In 2006, the Abuja Summit (African Development Bank Group 2006) for the region’s leaders set a fertilizer objective of 50 kg/ha for the region by 2015. Increased crop yields can lead to increased revenues for the farmer, which, in turn, can ultimately lead to a reduction of poverty. The return on investment (ROI) for farmers in SSA is quite high because of the low use of fertilizers; therefore, the marginal gains of fertilizer applications are higher. Several studies have shown that the ROI of $1 invested in fertilizers is high for maize growers in many SSA regions—for example, a
$4.5 \text{ ROI in Uganda (Jansen et al. 2013), $3.8 in Malawi (Ouattara et al. 2017), and $3.5 in Ghana (Essel et al. 2020). The ROI is even higher for beans, with returns averaging $6.3\text{-}12.3 \text{ per dollar invested in Rwanda, Tanzania, and Zambia (Kaizzi et al. 2018). }

Africa has the resources to achieve food self-sufficiency and even contribute to global food security because it has 60\% of the global arable lands and the raw materials and skills to supply fertilizers to its own farmers. Although nitrogen fertilizer production relies on capturing the nitrogen available in the air, phosphorus and potassium are nutrients that are mined, and 70\% of the world’s phosphate reserves are in Morocco (United States Geological Survey 2020) and are managed by OCP.

Yet, although African leaders set the objective of an average application of 50 kg/ha in 2006—a 2.5-fold increase of fertilizer use—OCP was unable to expand its production capacity because it was heavily indebted. In 2006, it was in such financial distress that it needed a massive cash injection to survive. The company held a $3.8 billion long-term liability and had generated mostly negative net income since 1999. Founded in 1920, OCP was heavily bureaucratic and siloed. The company had historically focused on maximizing the volume of ore extracted, which produced poor financial performance and limited its ability to grow. A new team led by Dr. Mostafa Terrab, who received his PhD in operations research at MIT, took leadership of the company in 2006. As chairman and chief executive officer, his mandate was to develop a new strategy, address the financial performance, improve the operating processes, deal with internal politics, and, perhaps most important, transform the culture of the organization. As a result of this leadership, OCP is now highly profitable ($2 billion earnings before interest, taxes, depreciation, and amortization (EBITDA) in 2020, although prices of fertilizers were near the bottom of the price cycle), its culture is oriented toward serving its end customers while optimizing value, and its sales and operations departments work collaboratively. Figure 1 shows an overview of OCP’s sales and supply chain.

In this paper, we outline how optimization played a key role in the transformation of OCP. The optimization model that OCP and Dynamic Ideas built is responsible for a cumulative financial impact of $2.3 billion in the period 2015–2020. This represents 23.2\% of the cumulative EBITDA. Increased profitability ultimately led to the launch of an unprecedented capital expenditures (CAPEX) program of $20 billion that resulted in the doubling of OCP’s mining capacity and the tripling of its fertilizer processing capacity. As a result, OCP generated an additional eight million tons of fertilizer production capacity in the past decade, thus improving the food security of the world.

The rest of this paper is structured as follows. In The Central Problem, we describe the central problem that OCP faced. In The Analytics Approach, we outline the optimization model the OCP and Dynamic Ideas team developed, and we discuss scalability, extensions, and its deployment in OCP. In Impact, we present the overall impact of the optimization model, and in Concluding Remarks, we summarize our conclusions.

The Central Problem
In this section, we describe the various decisions made by OCP sales and production managers to market products and produce the sales portfolio within the complex supply chain. Figure 2 shows key supply chain steps.
from the perspective of the overall value chain. The main components of OCP’s value chain are as follows:

Sales mix: OCP is a major contributor in the phosphates global trade. Sales span a wide range of products (43 grades of fertilizers, 9 qualities of phosphoric acid, and 22 grades of phosphate rock) and serve all regions in the world. The OCP sales team must determine which buyers to target with which products, in what quantities, and at what price. In addition, some opportunities consist of spot sales, whereas others require a long-term commitment, typically quarterly or yearly.

Extraction: Morocco’s phosphate deposits are sedimentary deposits. They consist of successive layers of different qualities of phosphate ore in addition to layers that do not include any phosphate. When extracting phosphate raw rock (ore), mine planners can choose to extract each layer separately, which results in more qualities available, or extract groups of layers simultaneously, which averages together the quality of the extracted layers but increases the volume extracted and lowers the cost, because operations are simpler.

Beneficiation: Beneficiation is the process of improving phosphate ore quality. It includes various techniques such as washing, flotation, and calcination in dedicated physical treatment facilities. Each extracted phosphate quality of raw rock can either bypass benefici- ation or undergo one or a combination of the five OCP beneficiation processes. The process increases the quality of phosphate ore but reduces its volume. Beneficiation decisions are a trade-off between volume, quality, and processing cost. Mining operators must determine what volume of each raw rock grade extracted requires beneficiation and what beneficiation steps it should undergo.

Blending and logistics: Miners must blend the various grades resulting from beneficiation to produce commercial grades that are exported or shipped to OCP chemical facilities. This step, which takes place in the physical treatment facilities, is critical because it determines the exact mixes that should be used to produce the mine’s final products. The planning must integrate the impact of time as a dimension because the availability of beneficiated raw rock varies over time. Careful planning of beneficiated raw rock inventory is critical to ensure the continuity of operations because for each commercial grade, the number of blending possibilities to be considered is limited.

Phosphoric acid production: Phosphoric acid production is the first chemical processing step. Phosphoric acid is made by mixing phosphate rock and sulfuric acid. At this stage, the phosphate rock input characteristics are important because (a) they significantly impact the production capacity and consumption of raw materials (i.e., phosphate rock and sulfuric acid), and (b) some specific acids can only be manufactured with a specific ore layer, which necessitates careful tracking of extracted ores along the value chain. OCP has 10 phosphoric acid lines that produce various concentrations of phosphoric acid depending on its intended use (i.e., export or fertilizer grade production).

Fertilizer production: OCP produces 43 grades of fertilizers in its 15 fertilizer production lines. Operators must schedule the production to match demand while
reducing downtime resulting from lines switching between grades to meet the sales schedule.

Intermediary stocks and logistics flows: Operators must manage stock at multiple intermediary storage facilities with specified capacities to avoid production stoppages as a result of materials shortages, production conflicts, or capacity constraints.

This entire process results in thousands of interdependent decisions that necessitate the strong alignment of all the operational units within OCP to achieve an optimal result. Timing is also a key element of the overall equation because decisions within the value chain have different corresponding planning horizons ranging from daily to annually. For example, although fertilizer production lines can be switched over during the month, raw materials sourcing has a longer lead time and must consider commercial negotiations, suppliers’ lead times, and the time needed for bulk shipping. Moreover, mine planning is a multiple-year exercise that requires significant effort for site preparation and to ensure the right machinery (e.g., draglines, shovels, trucks) is on-site.

Although these decisions are totally interdependent, prior to 2006, integration of the decision processes of the mining, chemical processing, and sales departments within OCP was limited. The sales department would have an idea about the capacity available and would then commit to a sales portfolio maximizing what that department perceived as the gross margin; it would typically give priority to products with the highest unitary gross margin. The mining department would aim to maximize the volume extracted while satisfying the rock exports order book. The chemical processing department would work with the rock quality resulting from mining operations to fulfill the demand expressed by the sales team. Information sharing among departments was limited because of the strong hierarchical structure and the natural conflicts that occur among production and sales entities. The mining and chemical processing departments had little visibility to the sales pipeline and would present their process as static to avoid being challenged on the way they conducted their operations. The end result was a process that contributed significantly to OCP’s financial losses at the time.

The Analytics Approach

Starting in 2006, OCP’s new management acknowledged that it had a number of managerial and operations issues to address. First, the production and sales departments needed to be aligned on the same objectives, profits, and customer satisfaction metrics. In addition, each department needed to have a better understanding of the impacts of each of its actions on the entire value chain, which implied more transparency. Finally, a deep culture shift was needed to sustain a drastic change in processes and spirit of cooperation.

To address these issues, OCP management created the “business steering” team to coordinate the efforts of all departments and align them on common objectives; however, a model spanning OCP operations was necessary to find improvement initiatives and convince senior management to implement these initiatives. When Dynamic Ideas and OCP began their collaboration in 2011, they built such a model capturing the thousands of decisions OCP managers needed to make to enlighten their decision-making process and optimize OCP sales and operations.

Today, Dynamic Ideas teams are involved in research and development phases, in the evolution of the optimization tool, and in testing. During the testing phases, both the OCP and Dynamic Ideas teams work together to disaggregate and understand the rationale of each recommendation. Dynamic Ideas teams are not involved in running and interpreting the operational runs of the model. This is done autonomously by OCP teams.

A High-Level Description of the Model

In what follows, we provide an overview of the model without mathematical details to give the reader insights on the model and its complexity. Although optimization models in the supply chain and in mining in particular have a long history (Graves et al. 1998, Graves and Tomlin 2003, Newman et al. 2010, Pimentel et al. 2010, Bodon et al. 2011, Newman and Weintraub 2014, Bouffard et al. 2018), the problem OCP faced had unique characteristics, and the formulation required a novel modeling approach. For example, (1) the problem had multiple objectives, (2) the solution allowed multiple ways to satisfy customer demand by allowing substitute products, and (3) the solution involved complex extraction, blending, transportation, and production constraints as well as nonlinear dynamics between OCP and its joint ventures (JVs). OCP sells phosphate rock to its JVs and has marketing agreements in place with the JVs to sell their products. The relationships are governed by shareholders’ agreements and other contractual arrangements, as is customarily the case. Each JV has its own governance bodies, a board of directors, and a managing director, all of whom make decisions within the limits of their powers and act in the best interest of the JV. Moreover, because several OCP departments would utilize the model, the modeling approach needed to be flexible to accommodate new requirements that the many constituents would add over time.

Objectives. The primary objective of the model is to maximize the margin on variable cost over the relevant time horizon. The margin is calculated as the revenue from direct sales to OCP clients plus the portion of revenue that OCP retains from sales by the company’s JVs minus variable costs. Variable costs include extraction,
physical and chemical treatment, transportation, and inventory, as well as the purchasing of raw materials needed for acid and fertilizer production. Secondary objectives include minimizing the line switching within the chemical facilities and maximizing the profitability of inventories. The model has provisions that allow for other primary objectives, such as minimizing cost, maximizing global or local production, and maximizing specific rock characteristics. Depending on the specific need, OCP uses one of these alternative primary objectives in the model.

The model starts with the extraction of different types of raw rocks at the mines and depicts the physical treatment process, where the beneficiation and the blending occur; the chemical facilities, where the production of acid and fertilizers take place; and the inventories of both marketable and intermediate products. It assumes that once a product arrives at the port, it can be used immediately to fulfill the demand.

**Constraints.** The constraints for the model used include the following:

- Minimum and maximum volume constraints on the client’s demand, where the demand is a vector of minimum demand, maximum demand, and price, and the model picks the optimal volume to satisfy. Moreover, the model can pick substitute products; for example, it can provide acid instead of fertilizers or a higher-quality rock in place of a lower-quality one.
- Extraction capacity limits, as all mines have a maximum extraction capacity. In addition, some mines must work at either full capacity or not at all; hence, we have binary “all-or-nothing” constraints.
- Capacity constraints at every stage of beneficiation.
- Treatment constraints that define the characteristics of intermediate rocks based on the beneficiation stages they undergo.
- Constraints on the blending of intermediate rocks.
- Constraints on logistics and transportation, including
  - on-site logistics, with examples including capacities of conveyor belts and secondary pipelines, and
  - long-distance transportation, in which a product travels to port using either rail or (for slurry) pipeline; each has constraints on the maximum quantities to be handled.
- Constraints on capacities of different stages of the chemical process within the chemical facilities and on ratios of the ingredients used in the production of acid and fertilizers.
- Constraints on the maximum number of rocks used in the chemical facilities.
- Constraints to capture the wasted capacity resulting from a chemical facility’s production line-switching products, which must be defined by binary variables.
- Constraints on loading for dry and wet finished products and unloading of raw materials (e.g., sulfur, sulfuric acid, ammonia).
- Minimum production constraints in all entities via binary constraints.
- Maximum-capacity constraints and minimum-buffer constraints on inventory levels plus the maximum number of products stored by inventory with binary constraints required.
- The dynamics between OCP and its JVs around tolling and the right to use production from other entities to satisfy specific demand contracts. Tolling is a process that allows different entities of OCP, including JVs, to maximize their production and asset utilization. It is an outsourcing of an intermediary production step between two entities of OCP.

**Decisions and Decision Parameters.** The key decisions that the model makes are as follows:

- Which contracts should OCP satisfy from a portfolio of client contracts that specify different qualities of rock, phosphoric acid, and fertilizers at different prices and at different times during the planning horizon?
- Which imported raw materials should OCP acquire using spot prices at the current market price or, alternatively, using established long-standing contracts, to be able to produce the necessary supply?
- What is the appropriate multiperiod detailed extraction, beneficiation, blending, chemical production, storage, and transportation plan to produce the final products?
- What ending inventory state (i.e., target value of inventory to have available at the end of the planning horizon) will allow OCP to optimally set up future production and demand estimates?

Some decisions need to be taken now (e.g., spot sales), some need to be planned a few weeks ahead (e.g., the chemical production plan), some must be planned at least one month ahead (e.g., new material sourcing), and some require a longer time frame (e.g., quarterly demand portfolio planning). We present a more detailed mathematical description of the model in the appendix.

**Data**

Appropriate data collection and cleansing are vital to build an optimization model of high fidelity. When OCP and Dynamic Ideas began collaborating in 2011, people, even those on the same team, could not agree on a uniform data set for the mining operations and the chemical processing. The OCP analytics team had to make a significant effort to collect and standardize the data. The modeling effort highlighted missing elements and inconsistencies in the data required for the model. Consequently, the team repeatedly iterated between improving the quality of data and updating the
modeling. The result was the development of a consistent database that OCP also uses for a variety of reports and key performance indicators, which is an indirect but significant benefit of the optimization modeling effort.

**Scalability**

We use the Gurobi engine to solve the problem both for its mixed-integer optimization (MIO) capabilities and for its ability to solve second-order cone-constraint problems. We use Gurobi’s threads parameter to run parallel optimizations in multiple-processor cores. (We found that increasing the number of threads above a specific threshold did not decrease run time even without competing processes or computer memory limitations.) A typical model run, without boat scheduling or extraction scenarios, for a monthly reoptimization during quarterly exercises includes approximately 6,500 rows, 6,000 columns (including 440 integer variables), and 20,000 nonzero elements for each month. If we run a multiperiod optimization, also without boat scheduling and without extraction scenarios, we have 18,000 rows, 16,000 variables (with 1,200 integers), and 60,000 nonzero elements. The optimization is nearly instantaneous, thus allowing the team to evaluate different assumptions, if needed. For example, if we do not consider the effects of switching product lines, and we perform a single-period optimization, the optimization requires 0.5 seconds for the optimization calculations and 31 seconds for a complete single-period run, which includes the output report generation. If we add the switching variables, the overall run time is still less than 1 minute.

In a daily mode with port loading, we have models with 600,000 rows and 500,000 columns (including 60,000 binary variables) and 2 million nonzero elements. In these cases, the optimization requires more run time and the optimization gaps are larger. We had to experiment with different formulations to identify those that yield tighter relaxations and with heuristics to create appropriate starting solutions so that Gurobi would avoid long delays at the root relaxation. For example, the starting solution may result from a preassignment of boats to gates or by running the optimization with less frequent time intervals. We also used callback functions to dynamically change the focus of Gurobi’s branch-and-bound algorithms toward feasibility or optimality and to change the fraction of time that Gurobi spends in heuristics to arrive at a near-optimal solution within an acceptable number of minutes.

**Deployment**

The first quantitative model, initiated in 2008 as a proof of concept, consisted of a single-period linear optimization model with one mine, one treatment plant, and one chemical facility, implemented in Excel. It included raw materials sourcing and the composition of client portfolios. The model illustrated the potential benefits of an integrated optimization approach. Dynamic Ideas became involved in 2010 to model a scalable, mixed-integer, multiperiod optimization model.

The team developed tailor-made Python code leveraging Gurobi algorithms for optimization purposes. Although components of the multiperiod optimization model were implemented in 2011, the model we outline in the section The Analytics Approach was implemented and deployed progressively. We deployed its fully functional version in 2015.

At the same time, we developed a digital collaborative platform to enable an increasing number of OCP managers, including some outside the business steering team, to understand and use the model to simulate the impact of their decisions. As different departments became familiar and comfortable with the new approach, this model became OCP’s primary decision-making platform. To accomplish this, OCP and Dynamic Ideas educated OCP’s managers and engineers at several multiple-day corporate retreats, ensuring that employees across the company had knowledge, appreciation, and competence with analytics. The solution is installed on a server that allows full accessibility to commercial, industrial, and supply chain teams. Standardization and simplification around a user experience were key to meeting monthly deadlines for the business steering committee (composed of executives), which oversees decision making and institutionalized the overall approach. The progressive step-by-step approach was instrumental in ensuring that the model grew in maturity while continuously maintaining credibility and acceptance throughout the company. It is now strongly rooted in the overall organization’s decision-making framework and represents a natural go-to platform for solving newly identified recurring patterns and future challenges.

Currently, the team has developed significant evolutions of the model, which are currently under testing, including the following functions.

**Extraction module:** Although the core model assumes specific extraction capacities of predefined raw rocks, the mines have multiple extraction zones, each with a number of reference layers. To capture the flexibility of the actual mining settings, we created a rich optimization model that includes machine scheduling and generates many possibilities for the raw rocks with which OCP starts production. In particular, the model recommends how to schedule machines to extract different layers (and depending on how many and which layers we extract together, we produce raw rocks with different characteristics), respecting the maximum number of machines available, the integer constraints of machine
scheduling, and the minimum buffer requirements between each layer.

**Boat scheduling**; We formulated another extension to the core model to include boat scheduling. This extension includes making decisions on when each boat will enter the port (after its arrival and before its target departure date); the gate to which we will allocate it, while respecting length constraints and loading availability; the order of products we will load (if more than one); and the entire path from hangar to conveyors to gate (with the appropriate flow constraints). It also formulates the delays that occur when a boat enters and leaves the dock and when the loading switches between products and the flow capacity that is “wasted.” Most of the decision variables are binary, and the dimensionality increases because the origin of production is important. Although this is a complex formulation, this extension is important in allowing OCP to control the entire path from mines to boats because loading delays are expensive and disruptive to the operations.

**Daily supply chain model**: We developed a daily tactical model.

**Robust supply chain model**: We extended the core supply chain formulation to account for the uncertainty in demand and capacity and utilized robust and adaptive optimization (Bertsimas and den Hertog 2021). The resulting optimization model scales to the same sizes as the deterministic optimization model.

**CAPEX model**: We utilized the model to guide CAPEX prioritization along the supply chain and leverage potential inherent flexibility while considering future commercial portfolio evolution.

OCP has two primary uses for the supply chain model:

- The core (and historic) business steering model is used ad hoc, monthly, yearly, and over multiple years. All runs use the same code and almost the same master data structure.
- New enhancements, as discussed in the preceding, are currently being tested; they either are built around an independent module (e.g., the extraction module) or leverage the existing platform while adding supply chain detailed representation (e.g., the daily model).

Recently, OCP explored various standard off-the-shelf solutions for end-to-end corporate planning and optimization and found significant gaps in comparison with the in-house customized model. For example, the company found:

- a lack of full coverage of the OCP value chain given specificities of each step (e.g., extraction, port management) and end-to-end business integration,
- difficulty in incorporating important features (e.g., blending) into the operating process, and
- a low level of agility to capture supply chain subtleties.

This experience demonstrated the importance of OCP's customized model if the company is to correctly address specific configurations to unlock value.

**Impact**

In this section, we outline the impact (i.e., primary and secondary benefits) of the optimization model. Primary benefits cover financial impact, transformation of the company culture, and impact on global food security. Secondary benefits relate to the impact on education and on other companies through its portability.

**Financial Impact**

We estimate the optimization model to be responsible for a cumulative financial impact of over $2.3 billion in the period spanning 2015–2020; this represents 23.6% of the cumulative EBITDA of $9.9 billion over this period. In each year from 2015 to 2020, the optimization model was key to generating at least 20% of OCP's annual EBITDA.

To justify this estimate, we calculated the financial benefit for the years 2017–2020. For 2015, we base our estimate on a report (covered by a confidentiality agreement) that a major consulting firm completed; for 2016, we estimated the financial impact by averaging the ratio of estimated impact over EBITDA for 2017–2020 and 2015. We looked at the supply chain, including all operations and demand, at that time, and we compared the revenue generated through the multiperiod comprehensive planning of the current model with the revenue that would have been generated under the siloed approach that the company followed before the transformation. Using that approach, the sales team followed a higher-margin-first heuristic in determining which contracts to satisfy first, and the operation teams in the mines and chemical facilities used their own heuristics to calculate the available capacity to satisfy the demands. Moreover, no manager within the organization had a clear view of the complete supply chain; therefore, managers missed small changes that could have unlocked revenue.

We therefore categorize the financial impacts into four groups: (1) sales portfolio arbitrage, (2) the multiperiod model effect, (3) capacity efficiency, and (4) improvement of the OCP supply chain’s design. In the remainder of this section, we present our results for 2015–2019 and provide details of our estimating process for 2020.

**Sales Portfolio Arbitrage**. Before the implementation of the business steering process, sales portfolio arbitrage was driven mainly by the prioritization of high
gross margin products. This led to suboptimal decisions. To illustrate, we compare the unit margin of different products in the OCP portfolio and their shadow prices of the supply chain flexibility (see Figure 3), which we extracted from a business steering committee presentation. Although the decision to prioritize a sales pipeline based on unit gross margin might intuitively seem like a sound choice, it would lead to missed opportunities that impact the bottom line.

When we compared the solution of the optimization model in a single-period mode for 2020 to a portfolio that favors higher unit margin products, which would have been the default policy in the absence of the model, we found an incremental benefit of approximately $200 million. We ran the optimization model in a single-period mode to separate the positive impact of the holistic view of the sales portfolio arbitrage that the model explores from the multiperiod optimization effect, which we address next. We also assume that the operations managers under the heuristic approach could use the optimization model to optimize their capacity and resolve their bottlenecks, again to separate the sales arbitrage effect from the capacity efficiency effect, which we quantify separately later in this section.

**The Multiperiod Model Effect.** Multiperiod planning is a key feature that unlocks value as a result of tactical planning of the sales pipeline and inventory optimization management. When we run the model for each period separately, without knowledge of the inputs for the foreseeable future, arbitrage opportunities are missed because closing deadlines for sales opportunities differ. For 2020, we compared running the optimization in a single-period mode (monthly), which, of course, overestimates the solution quality in the pre-optimization period, and we found that the solution the multiperiod model produced, which OCP implemented, is higher than the single-period solution by approximately $130 million.

**Capacity Efficiency.** Operations scheduling is key to preventing bottlenecks and enabling maximum-capacity utilization. Bottlenecks can occur in production lines, storage facility buffer stock, internal connections, transportation, and loading operations. A careful and optimal production schedule results in greater volumes to sell and thus increased EBITDA. Because the model determines the optimal scheduling of production lines, intermediary product flows, and overall resource allocation, it minimizes the downtime related to switching from one fertilizer grade to another, carefully utilizes storage capacity, and better integrates with the loading schedule of ships. In 2020, we compared the capacity induced by automatic optimized blending to manual

**Figure 3.** (Color online) The Graphs Compare Unit Gross Margin per Product with Shadow Price

Note. Global margin refers to sales minus variable cost; however, because fixed costs are static, optimizing global margin is equivalent to optimizing EBITDA.
manufacturing paths and found that the solution produced by the optimization model yielded an output increase of approximately 5%, which is equivalent to $60 million.

The overall supply chain has several bottlenecks that limit capacity beyond the blending management previously discussed. In particular, the chemical facilities might be limited in the production of specific products, and a locally optimal approach in storing and transportation, which was the strategy before OCP’s transformation, leaves room for improvement. The optimized tactical solution adjusted the commercial portfolio by finding additional opportunities in specific products that limited the main bottlenecks, exploited the maintenance schedules to further decrease the bottlenecks, and utilized nonconventional storage and transportation options. Our estimate of decreasing bottlenecks by tactical scheduling is approximately $60 million.

**Improvement of the OCP Supply Chain Design.** The final component of financial impact is the opportunity to improve the overall process. Each yearly, quarterly, and monthly business steering exercise of running the optimization model identifies the components of the supply chain that create bottlenecks. This exercise allows for redesign analysis, leading to the creation of links and nodes that further increase the flexibility of the supply chain and create even more value. The decisions are informed by a careful analysis of shadow values related to the capacity constraints and their recurrence over time. In 2020, establishing an additional intermediary product transfer pipeline and proactive storage management led to an EBITDA increase of approximately $40 million.

**Total Financial Impact.** Summing together all these additive financial contributions for 2020 demonstrates that the use of the optimization model resulted in an EBITDA increase of $490 million compared with the EBITDA generated preoptimization. This represents 24.0% of EBITDA for 2020. In Table 1, we record the EBITDA and the financial impact of the optimization model for the years 2017–2020, which we estimated using the same methodology we used for 2020. For 2015, our estimate is based on the above-mentioned confidential report that the major consulting firm completed and is consistent with our internal estimation process. For 2016, we estimated the average percentage of EBITDA from 2015 and 2017–2020 to be 23.6%, which when multiplied with the EBITDA for 2016 leads to an estimate of $308 million for the financial impact of the optimization model. This process leads to an estimate of over $2.3 billion of financial impact attributed to the optimization. To put this number into perspective, the cumulative EBITDA for 2015–2020 was $9.9 billion; therefore, our estimate of over $2.3 billion of financial impact represents 23.6% of EBITDA. Moreover, in each year from 2015 through 2020, the optimization model is responsible for generating at least 20% of the annual EBITDA.

The incremental profitability in the past decade has fueled OCP’s financing capacity. OCP is consequently delivering a $20 billion CAPEX program. Its first phase led to the doubling of OCP’s mining capacity and the tripling of fertilizer production capacity, resulting in OCP bringing an additional eight million tons of fertilizers production capacity in the past decade, thus significantly improving the food security of the world. The optimization model and OCP’s first wave of its $20 billion CAPEX program are intrinsically linked. In addition to contributing to OCP’s profitability, the optimization model uncovered recurring bottlenecks in the supply chain and underserved demand that led to a prioritization of the overall CAPEX program.

**Transformation of the OCP Culture**

Prior to the implementation of the business steering process, the mining, chemical processing, and sales departments were profoundly siloed. Each department had its own key performance indicators and was satisfied in being responsible for only the tasks it was directly operating; therefore, each department was unwilling to cooperate with other departments, especially because the benefits of change were hypothetical. In addition, risk aversion to consider such interdependent initiatives prevented departments from increasing any collaboration. OCP’s senior management needed to address those organizational issues to break down the silos and convince operators and managers to work together more closely.

The difficulty of transformation was exacerbated by OCP’s history as a century-old company with a strong, rigid, bureaucratic, and hierarchical management structure. The business steering group spent significant time with each department to build a model, which the main operational entities (i.e., departments)
within OCP collaborated in constructing. After integrating many of the major decisions involved in operations and sales contracts, the business steering group ran several iterations of the model, each differing depending on prices and capacity history. The results of the models showed missed opportunities of hundreds of millions of dollars annually.

Given the scale of this opportunity cost, departments were compelled to work closely with each other; no department wanted to be responsible for such huge losses. To persuade the organization to work together as a whole, senior management identified two main objectives. First, it had to create quick wins to convince department managers of the need. The business steering group identified a number of projects and initiatives, which would be implemented successively, to improve operations and the sales portfolio. Projects included changes in the sales mix that reduced the bottlenecks in the supply chain and changes in the design of the supply chain that increased production capacity with limited CAPEX. Second, the business steering group organized committees, consisting of people from the sales, mining, and chemical processing departments, which met quarterly to consider the key sales and operations strategies and then helped senior managers in their decision making and planning. These committees produced a unified quarterly production and sales plan across these departments.

As a result of organizing these committees and projects, the departmental teams progressively learned to work collaboratively and with a detailed understanding of the impacts across the business because they all used the optimization model to make decisions. Joint decisions were not based on qualitative discussions but on concrete mathematical calculations that targeted OCP’s overall margins rather than local optima.

As more managers became involved in the business steering process, additional details were introduced into the model to account for the high complexity of OCP’s supply chain and sales dynamics. This ultimately led to the continuous improvement of the model, its use by an ever-increasing number of managers, a greater scope within the model, and a wider implementation of its solutions. The model is currently being used to evaluate different courses of action on a monthly, quarterly, yearly, and multiple-year basis—and sometimes on an ad hoc basis. Users now include members of departments other than the business steering department (e.g., marketing, raw materials procurement). Managers in the mining and chemical processing departments do not rely only on the production plans the model produces but now use it to simulate and evaluate initiatives they are implementing. The optimization model not only gathered all operational decisions around a common platform, but we believe that it also enabled a process of continuous improvement through the involvement of an increasingly larger community within the company.

The centrality of the optimization model to OCP created a culture of collaboration and innovation among its departmental teams. This, in turn, has led to numerous improvements of the model and the addition of new capabilities that would not have been possible without the transformation in company culture accomplished through analytics.

Impact on Global Food Security

As we discuss in the Introduction, fertilizers play a key role in modern agriculture. They were irreplaceable ingredients in the Green Revolution, a “set of research technology transfer initiatives occurring between 1950 and the late 1960s, that increased agricultural production in parts of the world, beginning most markedly in the late 1960s” (Wikipedia 2021) that boosted food production. As a result of using technology, food production increased by 150% while the surface of forests and natural land, which was converted to crop production, increased by only 10% between 1961 and 2008. This helped many developing countries, including Mexico and India, evolve from their dependence on importing food to being able to export it. Yet most African countries still rely heavily on imports to feed their populations. One cause of this dependence on imported food is the productivity lag of SSA farmers. To achieve high productivity while protecting soil health, farmers need custom-made fertilizers adapted to their soil profiles and the crops they are growing. Since 2006, OCP has been heavily committed to improving the effectiveness of SSA farmers. As a result of customizing fertilizers for SSA farmers, OCP’s sales increased from 0.2 million tons in 2006 to 2 million tons in 2020.

OCP has been working with several soil and plant institutes, including the International Plant Names Index, the International Soil Reference and Information Centre, international universities, and African research institutes and transformation agencies such as the Ethiopian Agricultural Transformation Agency (ATA) to develop fertilizer formulas that are adapted to the specific needs of African farmers’ soils and crops. For example, OCP and the Bill & Melinda Gates Foundation supported the Ethiopian ATA in building a soil fertility map that ultimately led to the development of fertilizer formulas specifically adapted to the needs of Ethiopian farmers. Customized fertilizers developed as a result of this initiative increased corn yields by 37% while reducing the cost for farmers (see Figure 4). In only two years, this ultimately led the Ethiopian farmers to abandon generic formula fertilizers to adopt OCP’s nitrogen, phosphorus, and sulfur (NPS) fertilizers enriched with targeted micronutrients. OCP is also working with several other countries, such
as Nigeria, Ghana, Côte d’Ivoire, and Ethiopia, to propose tailored formulas. SSA is a vast region with multiple soil types. A one-solution-fits-all approach would be inefficient because soils, crops, and farmers have different needs. OCP then multiplied innovation initiatives to develop a customized formula for each region and crop in these countries.

The optimization model was at the heart of OCP’s African fertilizer initiatives. Specifically, it was instrumental in bringing more and better fertilizers to the continent (see Figure 5). The entire supply chain was empowered by the optimization engine. The company gained significant agility and operational efficiency in handling production switching and inventory losses. These new capabilities of handling production and supply chain variability in an environment with a volatile sales pipeline enabled OCP to customize the portfolio of fertilizers, which increased from 4 primary fertilizer formulas in 2007 to more than 40 formulas in 2020. The inherent complexity of handling operations would have heavily impacted margins and would likely have hindered OCP’s efforts to support SSA farmers with customized formulas.

The targeted investment recommendations of the optimization model resulted in an enhanced profitability that fueled OCP’s investment capabilities. The past decade has witnessed a doubling of the mining capacity and tripling of the chemical capacity, resulting in an additional eight million tons of fertilizer production capacity.

Impact on Education
OCP is a leader in the phosphate market, and part of its long-term mission is to contribute to the development of sustainable and resilient agriculture for the benefit of farmers worldwide.

In 2012, OCP launched the construction of the campus for the Mohammed VI Polytechnic University (UM6P), which is committed to training the continent’s future leaders in all areas of sustainable development, mining, and agricultural science. The university is focused on applied sciences and encouraging students to use data and analytics to inform strategy and decision making. It has partnered with MIT to explore research opportunities beyond OCP’s core business. As a developing continent, Africa is facing several challenges that require the use of analytics, operations research, and change management. In 2020, UM6P had 290 faculty members and welcomed more than 2,100 students from 23 African countries. Most of the programs taught at UM6P include analytics and operations research and stress the real-world successes of their use within OCP.

At MIT, the OCP MIO engine is taught as a case study of best practices within the Sloan School of Management. Professor Bertsimas and Dr. Terrab have taught a class, “LQ2: Leadership Through Quantitative and Qualitative Approaches,” for three years (2019–2021) as an elective for the MIT Sloan Executive Education program. The module outlines how analytics has led to the breaking down of existing silos and a culture of collaboration, innovation, and entrepreneurship across OCP. The class has been one of the most popular electives in the program; 100 students enrolled in 2021.

Impact on Other Companies Through Portability
The optimization model is portable to other companies and industries, as former OCP managers have
demonstrated. The potential value unlocked depends significantly on three factors: (1) the clear need for an end-to-end sales and operations plan, (2) complex supply chains that are subject to volatile business environments, and (3) the need for customized solutions (i.e., business environments similar to OCP’s in which off-the-shelf industrial solutions would not yield sufficient value). Former managers of OCP have successfully implemented a similar approach for a leading steel manufacturer that was suffering from profitability and indebtedness issues. Currently, other companies are asking OCP to share its experiences; for example, Morocco’s national airline is creating an optimization model with the assistance of UM6P, and a major agribusiness is evaluating the introduction of the model.

Concluding Remarks

Over the past decade, the optimization model has been a cornerstone of transformation at OCP. It has been expanded to include more details and a more holistic view of the supply chain and has gained acceptance throughout the company, from the mines to the ports, and at every level within the organization, from the executive leadership to people in the field. The commercial department, once skeptical of its value, now asks for feedback when creating its portfolio, and operations personnel are using it to schedule maintenance. Insights first uncovered by the model during the past decade are now part of the thinking at every step of the process. The model has become so embedded in OCP’s DNA that OCP’s leadership finds it hard to imagine implementing a strategy without using the model.

During the period 2015–2020, the optimization model was responsible for a cumulative financial impact of over $2.3 billion, which represents 23.6% of the cumulative EBITDA. The model is arguably the second-most important growth initiative during this period. Moreover, the most important initiative is a $20 billion CAPEX investment program (the first major program since the 1980s), which was shaped by the capacity requirements showcased by the model once the existing resources were optimized. Therefore, the financial impact of the optimization model cannot be overstated in highlighting OCP’s success.

Even in 2020, a challenging year for the global economy because of COVID-19, the optimization model helped OCP’s business steering committee to efficiently trade off imports and the production of sulfuric acid, choose the most efficient client demand portfolio to fulfill, select the most efficient products to produce, and reduce bottlenecks to its phosphorus pentoxide ($P_2O_5$) capacity. In addition, having a clear picture of the optimal production plan for the year helped the business steering group convince the sales team to postpone the fulfillment of NPK (a fortified fertilizer composed of nitrogen, phosphorus, and potassium) demand until later in the year and satisfy more demand for diammonium phosphate (DAP), a standard fertilizer, in the first quarter of 2020. The combination of these trade-offs and the insights are estimated to have had a $490 million impact on EBITDA in 2020 alone. Furthermore, the optimization model allowed OCP to produce customized fertilizers that have materially increased food production worldwide, especially in Africa, and have contributed to global food security.
Overall, this 10-year program has transformed OCP into an innovative, analytics-driven company, which is a strong ambassador for operations research and the management sciences.

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Appendix. The Optimization Model

In this section, we offer a summary of the mathematical modeling outlined previously in A High-Level Description of the Model. The model is a complex multiperiod MIO problem (Nemhauser and Wolsey 1988, Bertsimas and Tsitsiklis 1997, Bertsimas and Weismantel 2005). Our objective in this section is to give an overview, rather than a detailed description, of the variables and constraints we use. We define the most important variables and constraints, whereas we omit some of the specialized constraints: for example, we omit treatment constraints, some product ratio constraints, and constraints on connections and blending and variables associated with them; we do not list some of the min/max volume constraints; and we do not discuss nonlinear storage dynamics (i.e., involving second-order cone constraints) and JV dynamics in detail. The following are guiding principles for the formulation: (1) we need to model an arbitrary number and composition of products, a core element of OCP’s efforts to create the right product for the right customer and secure food stability in Africa and other countries worldwide, and (2) we need very general decision variables to allow the team to use the model to evaluate potential changes in the infrastructure such as introducing new connections between facilities, expanding the production capacities, or building additional inventories.

Contracts

Client contracts are defined by their primary product at a predefined price and the optimization determines how much of the demand to satisfy. The optimizer can also choose to substitute products: for example, it can provide, at different prices, acid to a client rather than fertilizer or a higher-quality rock rather than a lower one. In addition, minimum and maximum volume constraints are associated with both the primary and the substituted products. If we define $SD$ as the set of all contracts, the related primitive quantities for the model are

- $d$: a demand contract that OCP has with a customer, which specifies different qualities of rock, phosphoric acid, and fertilizers at different prices and at different times during the planning period, at which they can be satisfied;
- $p$: a product that is used in the OCP process; it represents raw material (e.g., ammonia), intermediate products (e.g., intermediate rocks in the beneficiation process), and final products (e.g., marketable rocks, fertilizers, and phosphoric acid); and
- $sp$: a substitution product that can be used to fulfill the obligations in a contract instead of the primary product.

We define the following decision variables associated with each contract:

- $sale(t,d,p)$: the volume of product $p$ sold for contract $d$ at time $t$, and
- $sub_sale(t,d,sp)$: the volume of substitution product $sp$ sold for contract $d$ at time $t$.

Figure A.1. (Color online) The OCP Northern Axis (AXE NORD) of the Supply Chain Shows a Connection That Links the Set of Chemical Facilities (EMA-PHOS, ..., JFC-IV) to a Set of Entities (Port Jorf or Truck Jorf)

Notes. MEA, DAoui, SC, and BA are the mines; BI is a drying plant. DWNSTRM, downstream platform; HS, headstation pipeline; JPH, Jorf Phosphate Hub.
Operational Modules of the Supply Chain
The primitive components of the supply chain are the entities (i.e., mines, treatment facilities, chemical facilities, storage facilities, ports, or logistical entities) and the connections, which are arcs that connect a set of entities to another set of entities; see the example in Figure A.1.

All those entities and connections are associated with products, from the raw rocks that are extracted in the mines, to the intermediate and marketable rocks that are transformed through beneficiation and blending in the treatment facilities, or to the acid and fertilizers of the chemical facilities. In addition, the supply chain uses raw materials (e.g., ammonia, potassium, or sulfur), which must be purchased for the production to take place.

We denote the set of entities by $S_E$, the set of connections by $S_C$, and the set of products by $S_p$. For each entity $e$, connection $c$, and product $p$, we define the following decision variables:

- $\text{input}(t,e,c,p)$: the flow of product $p$ over connection $c$ coming into entity $e$ at time $t$
- $\text{output}(t,e,c,p)$: the flow of product $p$ over connection $c$ coming out from entity $e$ at time $t$
- $\text{cost}(t,e)$: the variable cost of entity $e$ at time $t$; for example, the cost of a mine consists of the sum of extraction costs per quality extracted from each mine times the volume extracted

As we have already mentioned, the decision variables include the volume of raw materials OCP should buy in order to fulfill the optimal demand portfolio. OCP has two ways to acquire these volumes, either through outstanding contracts at a fixed price or in the open market at a spot price; therefore, we introduce the following variables for products $p$ in the set of raw materials, $S_{RM}$:

- $\text{import}(t,e,p)$: imported volume of raw material $p$ in the spot market at time $t$
- $\text{import}^c(t,e,p)$: imported volume of raw material $p$ through contract $c$ at time $t$
- $\text{cost}(t,e,p)$: the total cost of purchasing raw material $p$ at time $t$

For mines and ports, we also define additional decision variables:

- $\text{extract}(t,e,p)$: the extraction volume of product $p$ at mine $e$ and time $t$
- $\text{export}(t,e,g,p)$: the volume of product $p$ exported from gate $g$ of port $e$ at time $t$
- $\text{import}(t,e,g,p)$: the volume of product $p$ imported from gate $g$ of port $e$ at time $t$

Many of the output and input variables have maximum and minimum volume constraints that require a binary formulation. For example, if we define a binary variable:

- $\delta_{\text{prod}}(t,e,p)$, which is 1 if product $p$ is being extracted/produced at entity $e$ and time $t$,

we have

$$\min Vol(t,e,p) \cdot \delta_{\text{prod}}(t,e,p) \leq \text{extract}(t,e,p) \leq \delta_{\text{prod}}(t,e,p) \max Vol(t,e,p).$$

In addition, to decrease the overall complexity of operations, we want to bound the maximum number of products produced or entering an entity, again requiring binary variables for each product. For example, we bound the number of fertilizers produced in a chemical facility:

$$\sum_p \delta_{\text{prod}}(t,e,p) \leq \max No Products(t,e).$$

Dynamics of Physical and Chemical Treatment Facilities
The optimization model captures the dynamics of different entities, which for physical and chemical treatment facilities include a number of treatment options and a number of processing stages. Chemical treatments usually have multiple inputs, which are combined to make a single output product in a single stage, as the example in Figure A.2 shows. However, for physical treatments, the flow in beneficiation has one input (e.g., a raw rock), one output (that we denote as “intermediate rock”), and multiple stages (e.g., “Lavage Daoui” for washing, “Sechage Daoui” for drying).

For example, the flow denoted “Flow-SC-TBT (Tres basse teneur) + Lavage Daoui + Sechage COZ (Oued Zem)” takes one unit of raw rock “SC-TBT” as input and produces 0.58 units of an intermediate rock, which we denote as “SC-TBT + Lavage Daoui + Sechage COZ.”

For this, it consumes the following:

- 1 unit of the capacity of the stage “Lavage Daoui”
- 1 unit of the capacity of the stage “G1 → Lavage Daoui”
- 1 unit of the capacity of the stage “PEL → G2”
- 1 unit of the capacity of the stage “SC → PEL (Parc El Ouafi)”

Figure A.2. (Color online) The Example Shows Ammonia, ACP29, and ACP54 Being Combined in Chemical Treatment MP1 to Make Fertilizer
• 1 unit of the capacity of the stage “G2 → G1 (Goulotte (Gutter) 1)”
• 0.58 unit of the capacity of the stage “LoadingCOZ”

Moreover, physical treatments include blending units combining different flows to create intermediate or marketable rocks of different qualities. Consequently, we define the following variables:

- \( \text{flow}(t,e,r,f) \): the volume of flow \( f \) in facility \( e \) and treatment \( r \) at time \( t \), and
- \( \text{mix}(t,e,b,f_1,f_2) \): the volume of flow \( f_1 \) in facility \( e \) and blending unit \( b \) used to create flow \( f_2 \) at time \( t \)

In addition, because one of the key outputs of the model is the free capacity (or lack of it) at any production stage of the supply chain, we also define variables \( \text{extraCapa}(t,e,r,s,t) \) at time \( t \) at each stage \( s \) of the treatment unit \( r \) within entity \( e \).

The MIO models multiple operational constraints on the flows of different treatments that ensure that when flows are “blended” together, they create marketable rocks with the appropriate characteristics. Examples include constraints on the ratio of flows blended or transferred into different entities and maximum production constraints in the various treatments or stages (in addition to the entity-level maximum volume constraints). We note that the formulation permits many possibilities of mixing rocks, treated in different manners to ensure constraints on the maximum capacity and a maximum shared capacity. However, storage facilities are entities with connections to the rest of the supply chain and have complex dynamics. They have physical characteristics (i.e., length, width, and height) and a maximum capacity depending on the product stored, and they can store products from different origins, which arrive via the connections. However, for each product and each origin, a new stack is allocated within the storage unit, with the appropriate dividers between stacks. There are flow constraints on loading and emptying a storage facility, and frequently only one stack can be filled or emptied at a time. Products must stay stored for a minimum amount of time, and some products can only be stored in specific storage units. The formulation of the storage units is mostly through binary variables that capture whether a stack is empty, whether it is the last stack for JVs between OCP and other entities. Each JV is an entity and produces marketable products. Moreover, JVs have their own set of demand contracts, and the net revenue that the JVs produce is shared with OCP based on a shared-profit arrangement. The formulation captures the sourcing and revenue arrangements between OCP and the JVs, including tolling and the use of production from other entities to satisfy specific demand contracts. Those dynamics are nonlinear and are modeled through binary variables.

We also need to adjust the maximum capacity constraint for the switching:

\[
\text{extraCapa}(t,e,r,s,t) + \sum_{t \in \text{est}} \text{flow}(t,e,r,f) + \text{wastedCap}(t,e,r,s,t) = \text{Capa}(t,e,r,s,t).
\]

### Inventory Units and Storage Facilities

Inventory units are usually part of a specific entity and are associated with a set of products, each with a maximum capacity and a maximum shared capacity. However, storage facilities are entities with connections to the rest of the supply chain and have complex dynamics. They have physical characteristics (i.e., length, width, and height) and a maximum capacity depending on the product stored, and they can store products from different origins, which arrive via the connections. However, for each product and each origin, a new stack is allocated within the storage unit, with the appropriate dividers between stacks. There are flow constraints on loading and emptying a storage facility, and frequently only one stack can be filled or emptied at a time. Products must stay stored for a minimum amount of time, and some products can only be stored in specific storage units. The formulation of the storage units is mostly through binary variables that capture whether a stack is empty, whether it is the last stack of the hangar, and whether a specific product is stored at any time during the optimization, in addition to continuous variables that model variables such as the inflows and outflows, the starting and ending length of each stack, and the volumes. Similarly, the constraints enforce non-linear dynamics; for example, we can only have inflow of a single product from a set of connections, and in some cases, we cannot have both inflows and outflows to the same storage facility.

### Joint Ventures

In addition to the OCP entities, the formulation accounts for JVs between OCP and other entities. Each JV is an entity that has its own cost of operation; buys raw materials, rock, and acid, if needed, from OCP at predefined prices; and produces marketable products. Moreover, JVs have their own set of demand contracts, and the net revenue that the JVs produce is shared with OCP based on a shared-profit arrangement. The formulation captures the sourcing and revenue arrangements between OCP and the JVs, including tolling and the use of production from other entities to satisfy specific demand contracts. Those dynamics are nonlinear and are modeled through binary variables.

### The Objective Function

From the above-mentioned variables, we can define the objective function as

\[
\max \sum_t \left( \text{SCM_rev_total}(t) - \text{SCM_cost_total}(t) \right) + \text{Net_Rev_from_JVs}(t),
\]

where

\[
\text{SCM_rev_total}(t) = \text{Net_Rev_from_JVs}(t) + \sum_{\text{est}} \text{flow}(t,e,r,f)
\]

and

\[
\text{SCM_cost_total}(t) = \sum_{\text{est}} \text{switchCost}(t,e,r,s,t) + \text{wastedCap}(t,e,r,s,t)
\]

The objective function aims to maximize the net revenue from joint ventures while considering the operational constraints of the supply chain.
where the cost variable is defined as follows:

$$
\text{SCM\_cost\_total}(t) = \sum_{c \in C} \text{cost}(t, c) + \sum_{p \in \mathcal{P}} \sum_{c \in C} \text{cost}(t, p) + \sum_{c \in C} \left( \sum_{r \in R} \sum_{e \in E} \text{switchCost}(t, c, r, e) \right).
$$

and the revenue variable and net revenue from JVs are defined respectively as

$$
\text{SCM\_rev\_total}(t) = \sum_{d \in \mathcal{D}, s \in \mathcal{S}} \text{sale}(d, t, d, p) \cdot \text{Price}(d, p) + \sum_{d \in \mathcal{D}, s \in \mathcal{S}} \sum_{\text{sub}\_\text{sale}}(t, d, sp) \cdot \text{Price}(d, sp),
$$

$$
\text{Net\_Rev\_from\_JVs}(t) = \sum_{j \in \mathcal{J}} \text{JVrev}(t, j) \cdot \text{Profit\_Share}(j) + \sum_{j \in \mathcal{J}} \text{JVsourcecost}(t, j) \cdot \text{Profit\_Share}(j) - \sum_{j \in \mathcal{J}} \text{JVrevfromOCP}(t, j) - \sum_{j \in \mathcal{J}} \text{cost}(t, j) \cdot \text{Profit\_Share}(j).
$$

References


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Driss Lahlou Kitane is a PhD candidate at the Operations Research Center, MIT. His research focuses on sparse modeling using mixed integer optimization. He worked on applications of sparse regression and classification in spectroscopy and is currently working on methodological aspects including sparse principal component analysis. Prior to this, he assumed senior roles at OCP including marketing vice president, business steering vice president, and managing director.

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