Revealing Rival Marginal Offer Prices Via Inverse Optimization

Carlos Ruiz, Antonio J. Conejo, Fellow, IEEE, and Dimitris J. Bertsimas, Member, IEEE

Abstract—We consider a strategic producer that trades its energy in a multi-period network-constrained electricity pool and, for strategic reasons, is interested in identifying its rival producers’ offer prices. Considering industry practice, we assume that the strategic producer has knowledge of the daily market outcomes, i.e., energy quantities sold/bought and resulting locational marginal prices (LMPs) for each time period and all nodes of the network. Using this information we formulate an inverse optimization problem that allows estimating the rival producers’ offer prices that have been marginal at any of the time periods under study. Such problem is well-behaved, effectively identifies rival offer prices and can be efficiently solved. The effectiveness of the proposed technique is illustrated through a simple example and a realistic case study.

Index Terms—Electricity pool, inverse optimization, linear programming, marginal prices, market clearing.

I. INTRODUCTION

A. Background and Aim

We consider a strategic producer that sells electricity in a pool that is cleared using a continuous linear optimization based algorithm that incorporates a detailed representation of the transmission network, as in ISO New England [1] or PJM [2]. Market hourly outcomes per node, including accepted generation blocks for producers and accepted demand blocks for consumers, as well as clearing locational marginal prices (LMPs), are considered to be known to market agents some time after market clearing.

In order to elaborate its offering strategy, a key piece of information for the strategic producer considered in this paper is the knowledge of the marginal costs (or offering prices) of rival producers. However, production costs and offering prices of rival producers are proprietary information generally not available to the strategic producer.

Inverse optimization allows determining the “data” of an optimization problem given its solution, and is used in this paper to reveal the offering prices of rival producers, which constitute “data” for the clearing algorithm. In particular, the proposed procedure allows revealing those offering prices that have been marginal at some periods of time and therefore have an impact on the formation of the market outcomes, i.e., dispatched quantities and LMPs.

The problem addressed in this paper is of high practical significance for any strategic producer, as the knowledge of rival offer prices is a key component to define the strategy of the producer to achieve maximum profit in the market.

B. Approach

To generate appropriate market outcome data, consistent with known offering/bid (input) data, we formulate a market clearing model and simulate market clearing under a number of days with diverse load profiles. The result is market outcome data consistent with known input data.

Using the above market outcome data, the inverse problem of a strategic producer is formulated to estimate rival offer price data. Since such data is known, the quality of the revealing procedure can in turn be assessed.

Therefore, the proposed approach includes the following steps:

1) Offering data for all producers and bidding data for all consumers are assumed known.
2) Market clearing is simulated for a number of days with different load profiles and market outcome data gathered.
3) For the considered strategic producer, an inverse optimization problem is formulated and solved to reveal the offering prices of rival producers.
4) The price values obtained by solving the inverse optimization procedure are compared with the price data assumed in Step 1 above.

The technique proposed allows us to solve the problem addressed in a precise manner, i.e., rival offer prices are accurately revealed. On the other hand, the technique is highly efficient as it relies on solving a linear programming problem.

C. Literature Review and Contributions

To the best of our knowledge there are no references pertaining to electricity markets that use inverse optimization to reveal the price offers of rival producers. However, to some extent, the proposed technique is related to procedures for identifying inverse demand curves based on historical data as in [3]–[6]. However these procedures are generally based on time series analysis or clustering techniques and are approximate in nature. They conceptually differ from the inverse optimization approach proposed in this paper.

The contributions of this paper are threefold:

1) developing an efficient procedure based on inverse optimization to reveal offer prices of rival producers in a pool based electricity market;
2) formulating the procedure in 1) as a linear optimization problem that is easy to solve;
3) showing the accuracy of the proposed procedure through a realistic case study.

Inverse optimization techniques that allow finding the data of an optimization problem if its solution is known are described, for instance, in [7] and [8].

D. Paper Organization

This paper is organized as follows: Section II describes the market clearing model to generate market outcome data, Section III describes the inverse optimization model, Section IV is an illustrative example, Section V a case study, and Section VI provides relevant conclusions.

II. Market Clearing Model

A. Notation

The main notation used in this paper is stated below for easy reference.

Indices and Sets:

- $b$: Index for generation blocks running from 1 to $B$.
- $k$: Index for demand blocks running from 1 to $K$.
- $t$: Index for time periods running from 1 to $T$.
- $d$: Index for days running from 1 to $D$.
- $n/m$: Indices for nodes running from 1 to $N/M$.
- $\Theta_n$: Set of nodes adjacent to node $n$.

Parameters:

- $\lambda^{G}_{mnt}$: Price offer for power block $b$ of the strategic unit at node $n$ in time period $t$ on day $d$.
- $\lambda^{G}_{mnt}$: Price offer for power block $b$ of the rival unit at node $n$ in time period $t$ on day $d$.
- $\lambda^{\text{true}}_{mnt}$: Marginal cost of power block $b$ of the rival unit at node $n$.
- $\lambda^{\text{int}}_{mnt}$: Initial estimation of the marginal cost of power block $b$ of the rival unit at node $n$ in time period $t$ on day $d$.
- $\lambda^{P}_{mnt}$: Price bid of demand block $k$ at node $n$ in time period $t$ on day $d$.
- $P^{\text{max}}_{nb}$: Upper bound of power block $b$ of the strategic unit at node $n$.
- $P^{\text{max}}_{nb}$: Upper bound of power block $b$ of the rival unit at node $n$.
- $P^{\text{max}}_{nk}$: Upper bound of demand block $k$ at node $n$ in time period $t$ on day $d$.
- $\mu^{\text{G}}_{mnt}$: Total power produced by the strategic unit at node $n$ in the time period $(t = 0)$ prior to day $d$.
- $\mu^{\text{O}}_{mnt}$: Total power produced by the rival unit at node $n$ in the time period $(t = 0)$ prior to day $d$.
- $S_{nm}$: Susceptance of line $n - m$.
- $R^{\text{dwn}}_{mnt}$: Ramp-down limit of the strategic unit at node $n$.
- $R^{\text{up}}_{mnt}$: Ramp-up limit of the strategic unit at node $n$.
- $R^{\text{dwn}}_{mnt}$: Ramp-down limit of the rival unit at node $n$.
- $R^{\text{up}}_{mnt}$: Ramp-up limit of the rival unit at node $n$.

Variables for the Market Clearing Problem:

- $p^{G}_{mnt}$: Power produced by block $b$ of the strategic unit at node $n$ in time period $t$ on day $d$.
- $p^{O}_{mnt}$: Power produced by block $b$ by the rival unit at node $n$ in time period $t$ on day $d$.
- $p^{D}_{mnt}$: Power consumed by block $k$ of the demand at node $n$ in time period $t$ on day $d$.
- $\delta_{mnt}$: Voltage angle of node $n$ in time period $t$ on day $d$.
- $\lambda^{LMP}_{n}$: LMP at node $n$ in time period $t$ on day $d$ (dual variable).

Variable for the Inverse Optimization Problem:

- $\lambda^{\text{r}}_{mnt}$: Price offer of power block $b$ of the rival unit at node $n$ in time period $t$ on day $d$.

Regarding notation, the observations below are in order:
1) Missing subindices $d$ and $t$ in the constants above, indicates that these constants are identical for all $d$ and all $t$.
2) Note that some of the above variables/parameters can be considered as parameters/variables in the models below. This is conveniently specified throughout the paper.
3) For the sake of simplicity, we consider that at most one strategic unit, one rival unit and one demand are located at each node $n$.

B. Market Clearing: Primal Formulation

We consider an electricity auction in which producers and consumers submit their respective daily stepwise offer and bid curves to the independent system operator (ISO), who clears the market by setting the prices and the production/consumption quantities assigned to all market agents. This market clearing auction is formulated as a linear optimization problem that seeks maximizing the social welfare of the market. For day $d$, this auction has the form

$$\begin{align*}
\text{Minimize} & \quad \sum_{mnt} p^{G}_{mnt} + \sum_{mnt} p^{O}_{mnt} + \sum_{mnt} p^{D}_{mnt} \\
\text{subject to:} & \quad \sum_{mnt} \lambda^{G}_{mnt} p^{G}_{mnt} + \sum_{mnt} \lambda^{O}_{mnt} p^{O}_{mnt} - \sum_{mnt} \lambda^{D}_{mnt} p^{D}_{mnt} = \sum_{mnt} \lambda^{\text{int}}_{mnt} p^{D}_{mnt} \\
& \quad \sum_{mnt} \lambda^{G}_{mnt} p^{G}_{mnt} + \sum_{mnt} \lambda^{O}_{mnt} p^{O}_{mnt} - \sum_{mnt} \lambda^{D}_{mnt} p^{D}_{mnt} = \sum_{mnt} \lambda^{\text{true}}_{mnt} p^{D}_{mnt} \\
& \quad \sum_{mnt} \lambda^{G}_{mnt} p^{G}_{mnt} + \sum_{mnt} \lambda^{O}_{mnt} p^{O}_{mnt} - \sum_{mnt} \lambda^{D}_{mnt} p^{D}_{mnt} = \sum_{mnt} \lambda^{\text{int}}_{mnt} p^{D}_{mnt} \\
& \quad \sum_{mnt} \lambda^{G}_{mnt} p^{G}_{mnt} + \sum_{mnt} \lambda^{O}_{mnt} p^{O}_{mnt} - \sum_{mnt} \lambda^{D}_{mnt} p^{D}_{mnt} = \sum_{mnt} \lambda^{\text{true}}_{mnt} p^{D}_{mnt} \\
& \quad 0 \leq p^{\text{max}}_{mnt} \leq \sum_{mnt} \mu^{\text{G}}_{mnt} + \sum_{mnt} \mu^{\text{O}}_{mnt} \\
& \quad 0 \leq p^{\text{max}}_{mnt} \leq \sum_{mnt} \mu^{\text{G}}_{mnt} + \sum_{mnt} \mu^{\text{O}}_{mnt} \\
& \quad 0 \leq p^{\text{max}}_{mnt} \leq \sum_{mnt} \mu^{\text{G}}_{mnt} + \sum_{mnt} \mu^{\text{O}}_{mnt}
\end{align*}$$
Note that dual variables are indicated at their corresponding equations following a colon.

Problem (1) represents a market-clearing linear optimization based auction where the social welfare is maximized (the minus social welfare is minimized), as stated by the objective function (1a). The network is represented through a dc linear model that involves first and second Kirchhoff laws. The power balance at every node is enforced by (1b). Observe that the dual variable associated with each of the (1b) represents the hourly LMP for the corresponding node of the network. Generation and demand power bounds are imposed by constraints (1c)–(1e). Constraints (1f)–(1i) represent up and down ramping limits for the strategic and rival producers. Constraints (1j) represent the transmission capacity limits of each line. Phase angles for each node are bounded within the range $-\pi$ to $+\pi$ radians by constraints (1k). Constraints (1l) set $\pi$ as the reference node. The expression $m \in \Theta_n$ identifies the nodes $m$ adjacent to node $n$.

It should be mentioned that the considered electricity auction model (1) does not take into account discrete decisions variables so that its formulation is linear (and thus convex) and its dual problem can be easily derived. Therefore, problem (1) can be interpreted as an auction in which all the producers are indeed committed. Note that this is the case of most European markets, e.g., EEX [9], NordPool [10] or OMIE [11]. However, note also that the proposed model is not effective if non-convex unit commitment models are considered in which side payments are introduced to guarantee revenue adequacy [12]–[14].

### C. Market Clearing: Dual Formulation

Problem (1) is a linear optimization problem and thus its dual can be easily formulated as

$$\text{Maximize } \sum_{t,n,k} \sum_{m} \left( G_{\text{dtnk}} - G_{\text{dtnk}} \right) \mu_{\text{dtnk}} - \sum_{t,n,k} \sum_{m} \left( \mu_{\text{dtnk}} - G_{\text{dtnk}} \right)$$

subject to:

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} \mu_{\text{dtnk}} \leq 0 \quad \forall t, n$$

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} G_{\text{dtnk}} \leq 0 \quad \forall t, n$$

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} G_{\text{dtnk}} \leq 0 \quad \forall t, n$$

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} G_{\text{dtnk}} \leq 0 \quad \forall t, n$$

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} G_{\text{dtnk}} \leq 0 \quad \forall t, n$$

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} G_{\text{dtnk}} \leq 0 \quad \forall t, n$$

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} G_{\text{dtnk}} \leq 0 \quad \forall t, n$$

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} G_{\text{dtnk}} \leq 0 \quad \forall t, n$$

$$\sum_{t,n,k} \mu_{\text{dtnk}} - \sum_{t,n,k} G_{\text{dtnk}} \leq 0 \quad \forall t, n$$

where $\Xi \Xi \Xi \Xi \Xi$ is the set of optimization variables of problem (3). Note that $\Lambda$ is the set of dual variables.

Both problems (1) and (2) are used in the section below to formulate the inverse optimization problem that allows revealing the offer prices of rival producers.

### III. REVEALING RIVAL OFFER PRICES

The aim of the proposed procedure is to estimate $\lambda_{\text{dtn}}^{\text{dtn}}$ by knowing the market outcomes $G_{\text{dtn}}$, $\mu_{\text{dtn}}$, $P_{\text{dtn}}$, and the LMPs $\lambda_{\text{dtn}}$ that are dual variables.

To this end we formulate the inverse problem under the weighted $L_1$ norm [7].

#### A. Inverse Problem

Considering data for a number of days, the inverse problem under the weighted $L_1$ norm is

$$\text{Minimize } \sum_{t,n,k} \left| \lambda_{\text{dtn}}^{\text{dtn}} - \lambda_{\text{dtn}}^{\text{dtn}} \right|$$

subject to:

$$\lambda_{\text{dtn}}^{\text{dtn}} - \lambda_{\text{dtn}}^{\text{dtn}} + \mu_{\text{dtn}}^{\text{dtn}} = 0 \quad \forall t, n, b$$

$$\lambda_{\text{dtn}}^{\text{dtn}} - \lambda_{\text{dtn}}^{\text{dtn}} + \mu_{\text{dtn}}^{\text{dtn}} = 0 \quad \forall t, n, b$$

$$\lambda_{\text{dtn}}^{\text{dtn}} - \lambda_{\text{dtn}}^{\text{dtn}} - \lambda_{\text{dtn}}^{\text{dtn}} = 0 \quad \forall t, n, b$$

$$\lambda_{\text{dtn}}^{\text{dtn}} + \lambda_{\text{dtn}}^{\text{dtn}} = 0 \quad \forall t, n, b$$

$$\sum_{t,n,k} \lambda_{\text{dtn}}^{\text{dtn}} - \lambda_{\text{dtn}}^{\text{dtn}} + \sum_{t,n,k} \lambda_{\text{dtn}}^{\text{dtn}} - \lambda_{\text{dtn}}^{\text{dtn}} = 0 \quad \forall t, n, b$$

where $\Xi \Xi \Xi \Xi \Xi$ is the set of optimization variables of problem (3). Note that $\Lambda$ is the set of dual variables.
includes the variables associated with problem (2), i.e., $\Xi_d, \forall d$, plus $\lambda_{dtmb}^G, \forall d, t, n, b$; but not $\lambda_{dtmb}, \forall d, t, n$.

In short, the optimization variables of problem (3) that is solved by the strategic producer include the offer prices of all the units of rival producers ($\lambda_{dtmb}^G, \forall d, t, n, b$) and a number of auxiliary dual variables specified in set $\Delta$.

Observe that problem (3) minimizes

$$\sum_{dtmb} (\lambda_{dtmb}^G - \lambda_{dtmb}^{Gini})$$

(4)

over all feasible rivals’ offer prices $\lambda_{dtmb}^G, \forall d, t, n, b$, represented by conditions (3b)–(3c), where $\lambda_{dtmb}^{Gini}$ is an initial guess of the price offered by the rival producer at node $n$ for energy block $b$ in time $t$ of day $d$.

Constraints (3b)–(3c) are the optimality conditions of problem (1). Note that, contrary to [7], these conditions are not based on the Karush-Kuhn-Tucker (KKT) conditions. Instead, we take advantage of the linearity of problem (1) and incorporates a primal-dual formulation of the optimality conditions, which enforces:

1) primal constraints (1b)–(1l) [not included in (3) because they are already met, i.e., the market outcomes $P_{dtmb}^G$, $P_{dtn}^L$, $P_{dtb}^D$, and $\lambda_{dtmb}$ are directly obtained from problem (1)];

2) dual constraints (3c) [equivalent to (2b)–(2h)];

3) the strong duality theorem (3b) that stands for the equality of the primal (1a) and dual (2a) objective functions.

This primal-dual formulation, equivalent to the KKT conditions, is computationally efficient for linear optimization problems since it avoids the use of the nonconvex complementarity conditions [15].

Moreover, note that since problem (1) is linear (and thus convex), its associated Karush-Kuhn-Tucker conditions (equivalent to the primal-dual formulation presented in the paper) are both necessary and sufficient conditions for optimality. Additionally, the linearity of problem (1) also guarantees that regularity conditions are met at its optimal solutions and thus, under non-degeneracy assumptions, its associated multipliers are unique.

Finally, note that problem (3) decomposes by day $d$, i.e., the problem of each day $d$ is independent of the problem of other days, and thus, problem (3) can be solved in a decomposed manner.

B. Inverse Problem Assumptions

Problem (3) assumes perfect knowledge of the constants described below:

1) The technical parameters of each unit $n$ of the strategic producer, namely, capacity of each production block $b$ ($P_{nmb}^{Gmax}$), initial production level for each day $d$ considered ($P_{nmb}^{Ginc}$), and ramping-up ($P_{nmb}^{Gup}$) and ramping-down ($P_{nmb}^{Gdown}$) limits; and the cost parameters ($\lambda_{dtmb}^G$) of these units. Note that these data are known to the strategic producer.

2) The technical parameters of each unit $n$ owned by rival producers, namely, capacity of each production block $b$ ($P_{nmb}^{Gmax}$), ramping-up ($P_{nmb}^{Gup}$) and ramping-down ($P_{nmb}^{Gdown}$) limits, and initial production level for each day $d$ considered ($P_{nmb}^{Gini}$). The capacity and the ramping limits are not directly known by the strategic producer but they can be easily obtained from publicly available data of electricity production units. The initial production is easily obtained from the market outcome data.

3) The accepted production data including the production of all units of the strategic producer ($P_{dtn}^G, \forall d, t, n, b$), of all the unit of the rival producers ($P_{dtmb}^G, \forall d, t, n, b$), and the accepted demand data ($P_{dtb}^D, \forall d, t, n, k$) are obtained from the market data. Additionally, clearing price data including all LMPs ($\lambda_{dtmb}, \forall d, t, n$) that are also obtained from the market outcome data.

4) The upper bound of each demand block ($P_{dtb}^{Dmax}, \forall d, t, n, k$) and its associated price bid data ($\lambda_{dtb}^D, \forall d, t, n, k$) that can be considered equal to a large enough constant (if no additional information is available) or to the price cap of the market under consideration (e.g., $180/MWh in the market of the Iberian Peninsula [11]).

5) Transmission data including the susceptance of each transmission line. These data are generally available from the transmission operator and easily obtained by the strategic producer.

Additionally, in the case in which some of the technical parameters associated with the rival producers and the demand are not easily available, such as the size of the generation blocks ($P_{nmb}^{Gmax}, \forall n, m$) and the demand blocks ($P_{dtn}^{Dmax}, \forall d, t, n, k$), the screening step to “scan” rival offers needs to be specified by assigning a size to these power blocks that should be small enough to accurately identify rival offer prices, e.g., $P_{dtn}^{max} = P_{dtb}^{Dmax} = 1 MW$. Needless to say, price bid data ($\lambda_{dtb}^D, \forall d, t, n, k$) need to be consistent with these demand blocks in order to guarantee the feasibility of the inverse problem (3).

This paper considers that the accepted production data $P_{dtn}^{G/O}$ as well as the locational marginal prices $\lambda_{dtmb}$ are available to all market participants. Thus, the application of the proposed methodology is limited to those markets in which this information is public, such as EEX [9], Nordpool [10] or OMIE [11].

Additionally, in this paper we assume that the strategic behavior of the production units can only be exercised by altering their offer curves. We are aware of the limitation of this assumption but we believe that, although it may conceal some of the gaming capabilities of real-world producers, it still captures their main strategic tool, which is deviating the actual offer curve from the corresponding true marginal cost curve.

Note that only if the above assumptions are met, problem (2) is the dual problem of (1) and the strong duality theorem (3b) holds.

Nonlinear problem (3) is transformed below into a linear optimization problem.

C. Linear Formulation

Problem (3) is equivalent to the following linear optimization problem:

Minimize $\sum_{dtmb} (\alpha_{dtn} + \beta_{dtn})$

(5a)
subject to:
\[
\begin{align*}
\lambda^{O}_{dtnb} - \lambda^{Q}_{dtnb} - \alpha_{dtnb} - \beta_{dtnb} & \quad \forall d, t, n, b \quad (5b) \\
\alpha_{dtnb}, \beta_{dtnb} & \geq 0 \quad \forall d, t, n, b \quad (5c) \\
(3b) - (3c). & \quad (5d)
\end{align*}
\]

The solution of this problem, which will be referred to as \(\lambda^{O}_{dtnb}\), provides estimates for the offer prices of rival producers, \(\lambda^{O}_{dtnb}, \forall d, t, n, b\).

Observe that the estimates of the offer prices \(\lambda^{O}_{dtnb}\) corresponding to those blocks that have been marginal are not affected by the election of the initial guesses \(\lambda^{Q}_{dtnb}\). This can be seen by analyzing the system of (3b)–(3c) in which the value of each marginal energy block is uniquely determined. In other words, the market outcomes provided by (1) for each time period and network location can only be generated by one specific marginal block.

However, for a specific time period and network location, the blocks that are non-marginal, i.e., the blocks that do not set the market price, can adopt any value without altering the market outcomes. In this case problem (5) will force the estimates of these blocks to be equal to the initial guesses \(\lambda^{Q}_{dtnb}\). This can be illustrated as follows. Consider that an energy block \(H_{dtnb}\), which is always dispatched, is never marginal because it is offered at a price \(\lambda^{O}_{dtnb}\) that is always lower than the corresponding LMP \(\lambda_{dtn}\). Note that this block could have been offered at lower prices, i.e., \(\lambda_{dtnb} - \varepsilon\) with \(\varepsilon \in [0, \lambda_{dtnb}]\), without altering any of the LMPs and dispatched quantities resulting from the auction. This renders that the same market outcomes, the ones that are used as parameters in problem (5), could have been generated by infinitely many non-marginal offers \(\lambda_{dtnb}\) and therefore these non-marginal offers cannot be determined. Note that a similar reasoning can be applied if the non-marginal block under consideration is always offered at a price higher that its corresponding LMP.

**D. Estimating Rival Producers’ Marginal Costs**

We assume that each producer offers its energy at prices that are equal to or higher than its actual marginal costs (to avoid financial losses), [8]. Taking this into account, we use a simple rule to estimate each rival producer marginal cost curve, \(\lambda^{O}_{nb}\) at each node, \(\forall b\), which is based on the joint consideration of all the daily offers prices provided by the solution of problem (5), i.e., \(\lambda^{O}_{dtnb}\). The proposed rule is as follows:

\[
\hat{\lambda}^{O}_{nb} = \min_{\forall d, t} \{ \lambda^{O}_{dtnb} \} \quad \forall n, b. \quad (6)
\]

Note that (6) estimates the marginal cost associated with an energy block, \(\lambda^{O}_{nb}\), as the minimum price offered for it over the days \(d\) and time periods \(t\) considered.

It should be mentioned that other assumptions can be made regarding the offering strategy of the rival producers. For instance, it can be considered that, for strategic reasons [15]–[17], producers may offer below their true marginal costs. In this case, other rules different than (6) can be used to estimate each rival producer marginal costs, e.g., to compute each \(\hat{\lambda}^{O}_{nb}\) as the average off all \(\lambda^{O}_{dtnb}\), \(\forall d, t\).

The proposed methodology is of especial interest to those producers that have a significant market share and therefore are able to exercise their market power. Note that a small producer (with few generating units) may not be interested in the proposed methodology, since, due to its size, its best strategy is generally offering at its marginal costs. For this reason, for the numerical experiments we consider that the strategic producer owns a significant number of generating units in comparison to its rival producers. Moreover, this oligopolistic market structure is present in many electricity markets worldwide.

Needless to say, the proposed model can be also used by small producers. In this case, the number of offer prices to estimate \(\lambda^{O}_{dtnb}\) is high in comparison to the number of offer prices known by the small producer \(\lambda^{G}_{dtnb}\). Therefore, in order not to worsen the accuracy of the estimation of the rival offer prices, it would be necessary to expand the time horizon under analysis. Note that in doing so the number of available locational marginal prices \(\lambda_{dtn}\) that are used in (5) to identify the marginal blocks of the rival offer curves increases.

**IV. ILLUSTRATIVE EXAMPLE**

**A. Data**

In this illustrative example, we analyze the three-node test system depicted in Fig. 1. For simplicity, we consider that there is one demand, one strategic unit and one rival unit at each node of the network. Additionally, all transmission lines are considered to be equal and characterized by a susceptance \(B_{nm} = 1000\) p.u. and a capacity \(P_{nm}^{\max} = 100\) MW.

Table I provides the rival producers’ marginal costs curves where all power blocks are considered of the same size and equal to 100 MW. Observe that these are the marginal costs curves to be identified by using the inverse problem (5) and the rule (6). For clarity, we assume that all rival units have the same ramp-up and ramp-down limits i.e., \(R_{dwn}^{nb} = R_{dwn}^{h} = 1000\) MW/h.

We assume that rival producers offer each one of their power blocks at a price that is equal to or higher than its actual marginal cost. For simplicity, in this illustrative example we consider that the rival offer curves for each day \(d\) and time period \(t\) are randomly generated as follows:

\[
\lambda^{O}_{dtn} = \lambda^{O}_{nb} \times (1 + 0.1 \times \Omega_{dt}) \quad \forall d, t \quad (7)
\]

where \(\Omega_{dt}\) is a random value obtained from an uniform distribution on the interval \([0, 1]\). To illustrate this offer curve formation, Fig. 2 depicts the marginal cost curve (black line) as well...
as the randomly generated offer curves (grey lines) for a particular rival producer (the one located at node 1) and for different days and time periods. Observe that all the generated prices are above their corresponding marginal costs.

The strategic producer’s offer curves are provided in Table II where, for simplicity, they are assumed to be identical for all days, periods of time and network nodes. Similarly, all power blocks are considered of the same size and equal to 100 MW while the ramp-up and ramp-down limits are 1000 MW/h.

For the sake of simplicity, all the production units are assumed to be offline at $t = 0$, i.e., $P^{G, ni}_{dn} = P^{G, ni}_{dn} = 0$ \forall d, n.

Fig. 3 shows the demand curves for each node of the network. Observe that most of the demand is located at node 3 and thus an energy flow from nodes 1 and 2 to node 3 is expected.

The demand bid curve for each day and hour is obtained by multiplying the capacity of each energy block in Fig. 3 by the factors depicted in Fig. 4. Note that three days are represented, corresponding to peak, shoulder and base demand levels. Note also that each day is divided into 24 hours.

### B. Obtaining Market Outcomes

To obtain the energy quantities $P_{dn}^{G, k}$, $P_{dn}^{G, k}$, $P_{dn}^{D, k}$, $t, d, n, k$ and the LMPs $\lambda_{dn}, \forall d, t, n, k$ resulting from the market clearing auction, which are used by the strategic producer to identify its rivals’ offer prices, we run model (1) with the generation, demand and network data described above. Fig. 5 provides the resulting LMPs for the considered peak day [subplot 5(a)], shoulder day [subplot 5(b)] and base day [subplot 5(c)]. Due to congestion, prices are different across the network, being comparatively higher at node 3, which is where there is a comparatively higher demand level. Note that peak-day prices [subplot 5(a)] are also comparatively higher due to comparatively higher demand levels.

### C. Revealing Offer Prices

Taking into account the data provided in the previous subsection and considering initial guesses for the rival offer curves $\lambda^{G, ini}_{dn} = 0$, $\forall d, t, n, k$, we use the linear inverse problem (5) and the rule (6) to estimate the rival producers’ offer curves, i.e.,
Fig. 6. Estimated (solid line) and true (dash line) marginal costs. (a) Node 1. (b) Node 2. (c) Node 3.

\( \lambda^O_{d,n,b} \) ∀\( d,t,n,b \), and the rival producers\' marginal costs curves, i.e., \( \lambda^O_{n,b} \), ∀\( n,b \), per node.

Fig. 6(a)–(c) provides \( \lambda^O_{d,n,b} \) (grey lines), \( \lambda^{O\text{true}}_{n,b} \) (solid black line) and \( \lambda^{O\text{true}}_{n} \) (dash black line) for each node of the network. Observe that considering the inverse problem (5) and the rule (6) we can reveal most of the prices offered for each energy block and estimate their associated marginal cost, respectively. In particular, the model can reveal the prices of those blocks that have been marginal, i.e., that have set the LMP in any of the times periods and days considered.

Finally, note that at node 3 the fifth price block is never marginal and thus it cannot be estimated. However, simple heuristics can be used to complete the offering curve of rival units at node 3. One possibility is to identify the blocks that have not been estimated and assign them random prices until the resulting offer curve is increasing in price. To speed this process, these prices should be generated within a range between the estimated prices of the closest energy blocks of lower and greater energy levels than the one considered.

The prices depicted in Fig. 6(a)–(c) are provided in Table III for reference. This table also includes \( \lambda^{O\text{max}}_{d,n,b} \), which is defined as the maximum price that has been offered for each energy block over all days and time periods \( t \) considered, i.e., \( \lambda^{O\text{max}}_{n,b} = \max_{d,t} \{ \lambda^O_{d,n,b} \} \) ∀\( n,b \). Observe that all the estimates for the offer prices of rival producers (\( \lambda^O_{d,n,b} \)) lay within the interval \([ \tilde{\lambda}^{O\text{true}}_{n,b}, \tilde{\lambda}^{O\text{max}}_{n,b} ]\). It is worth mentioning that the knowledge of these price intervals is relevant for the implementation of robust optimization techniques [18] such as the one used in [19] to derive offering curves.

V. IEEE RTS CASE STUDY

In this section a realistic case study is analyzed. We consider the market clearing outcomes corresponding to a 24-period pool-based electricity market during one month (30 days). The system considered is based on the 24-node IEEE Reliability Test System (RTS) [20].

A. Data

Table IV contains data regarding the different types of generating units, one per column. The second row provides the power capacity of each unit, which is divided in 8 power blocks (rows 3 to 10) with their associated marginal production costs (rows 11 to 18). The last two rows contain the ramp-up and ramp-down limits.

Table V indicates how the different types of generating units, presented in Table IV, are distributed throughout the network and whether they belong to the strategic producer or to its rival producers.

Note that the marginal costs corresponding to the rival units are to be identified by using the inverse problem (5) and the rule (6).

We consider that, for each day \( d \) and period of time \( t \), the rival producers offer each one of their power blocks at a price that is equal to or higher than its actual marginal cost, following the price generating rule described in (7).

All the production units are assumed to be off at \( d = 1 \) and \( t = 0 \), i.e., \( P_{d,n}^{G\text{ini}} = P_{d,n}^{Q\text{ini}} = 0 \) ∀\( n \). Additionally, we consider that the initial production level of each generating unit at the beginning of each day \( d > 1 \) is equal to its total production at time period \( t = 24 \) of the previous day \( d - 1 \), i.e., \( P_{d,n}^{G\text{ini}} = \sum_b P_{d-1}^G \{ \sum_{t=24}^{t=1} t_b(t=24) \} \forall d > 1,n \), and \( P_{d,n}^{Q\text{ini}} = \sum_b P_{d-1}^Q \{ \sum_{t=24}^{t=1} t_b(t=24) \} \forall d > 1,n \).

Table VI provides data for the total system demand that is modeled by 10 power blocks (first row) with their associated marginal utilities (third row). The total demand is distributed throughout the nodes of the network as indicated in Table VII.

Additionally, to simulate the hourly and daily pattern of the demand, we assume that \( P_{d,n,b}^{D\text{max}} \) i.e., the upper bound of demand block \( k \) at node \( n \) in period \( t \) on day \( d \), is obtained as

\[
\phi \left( \tilde{\lambda}^{O\text{true}}_{n,b}, \tilde{\lambda}^{O\text{max}}_{n,b} \right)
\]

TABLE III

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
<th>( 6 )</th>
<th>( 7 )</th>
<th>( 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda^{O\text{true}}_{n,b} )</td>
<td>( 12 )</td>
<td>( 17 )</td>
<td>( 20 )</td>
<td>( 24 )</td>
<td>( 29 )</td>
<td>( 33 )</td>
<td>( 41 )</td>
<td>( 47 )</td>
</tr>
<tr>
<td>( \lambda^{O\text{max}}_{n,b} )</td>
<td>( - )</td>
<td>( 17.49 )</td>
<td>( 20.29 )</td>
<td>( 24.42 )</td>
<td>( 29.39 )</td>
<td>( - )</td>
<td>( - )</td>
<td>( - )</td>
</tr>
<tr>
<td>( \tilde{\lambda}^{O\text{true}}_{n,b} )</td>
<td>( -18.29 )</td>
<td>( 21.85 )</td>
<td>( 25.85 )</td>
<td>( 30.39 )</td>
<td>( - )</td>
<td>( - )</td>
<td>( - )</td>
<td>( - )</td>
</tr>
</tbody>
</table>

\[
\sum_b P_{d,n}^G(t=24) \forall d > 1,n \]

\[
\sum_b P_{d,n}^Q(t=24) \forall d > 1,n \]

Authorized licensed use limited to: MIT Libraries. Downloaded on April 20,2022 at 14:15:00 UTC from IEEE Xplore. Restrictions apply.
TABLE IV
IEEE RTS. TYPE AND DATA FOR THE GENERATING UNITS. “O/G” INDICATES EITHER RIVAL (O) OR STRATEGIC(G) (MW)

<table>
<thead>
<tr>
<th>Unit Type</th>
<th>oil</th>
<th>oil</th>
<th>hydro</th>
<th>coal</th>
<th>oil</th>
<th>coal</th>
<th>oil</th>
<th>coal</th>
<th>nuclear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit1</td>
<td>12</td>
<td>20</td>
<td>50</td>
<td>76</td>
<td>100</td>
<td>155</td>
<td>197</td>
<td>350</td>
<td>400</td>
</tr>
<tr>
<td>P_{O/G}^M</td>
<td>1.2</td>
<td>7.8</td>
<td>7.15</td>
<td>12.5</td>
<td>30</td>
<td>35</td>
<td>70</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>P_{O/G}^{M\max}</td>
<td>1.2</td>
<td>8</td>
<td>7.25</td>
<td>12.5</td>
<td>25</td>
<td>34</td>
<td>70</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>P_{O/G}^{M\min}</td>
<td>1.7</td>
<td>0.6</td>
<td>6.25</td>
<td>12.5</td>
<td>19</td>
<td>24</td>
<td>48</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

TABLE V
IEEE RTS. LOCATION AND TYPE OF GENERATING UNITS

<table>
<thead>
<tr>
<th>Strategic units</th>
<th>Type</th>
<th>76</th>
<th>76</th>
<th>100</th>
<th>155</th>
<th>400</th>
<th>155</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rival units</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>13</td>
<td>21</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

TABLE VI
IEEE RTS. DATA FOR THE TOTAL DEMAND

<table>
<thead>
<tr>
<th>k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_{D_{nb}}^{M\max} [MW]</td>
<td>800</td>
<td>200</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>\lambda_{D_{nb}}^{M} [$/MWh]</td>
<td>35</td>
<td>32</td>
<td>30</td>
<td>27</td>
<td>25</td>
<td>23</td>
<td>20</td>
<td>18</td>
<td>12</td>
<td>5</td>
</tr>
</tbody>
</table>

TABLE VII
IEEE RTS. LOCATION AND DISTRIBUTION OF THE DEMAND

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>% load</td>
<td>3.8</td>
<td>3.4</td>
<td>6.3</td>
<td>2.6</td>
<td>2.5</td>
<td>4.8</td>
<td>4.4</td>
<td>6.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Node</td>
<td>10</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>% load</td>
<td>6.8</td>
<td>9.3</td>
<td>5.8</td>
<td>10.1</td>
<td>3.5</td>
<td>11.7</td>
<td>6.4</td>
<td>6.5</td>
<td></td>
</tr>
</tbody>
</table>

B. Revealing Rival Offer Prices
Considering the data provided in the previous subsection, we simulate 30 market clearing auctions (1), one per day, each involving 24 time periods, which results in market outcomes \( P_{D_{nb}}^{M}, P_{D_{nb}}^{M\max}, t, n, k \) and LMPs \( \lambda_{D_{nb}}, \forall d, t, n \). These results are considered as parameters in the linear inverse problem (5) in order to obtain the rival producers’ offer curves, i.e., \( \lambda_{O_{nb}}^{M}, \forall d, t, n \). Again, we consider \( \lambda_{O_{nb}}^{M} = 0, \forall d, t, n \) (the initial guesses for the rival offer curves equal to zero). Finally, we apply the rule (6) to estimate the rival producers’ marginal costs curves, i.e., \( \lambda_{O_{nb}}^{M}, \forall n \).

VI. CONCLUSION
Using as data the market clearing outcomes for a number of days and solving the proposed inverse optimization problem allows estimating both the offer prices of rival producers and their associated marginal costs. Line congestion results in different LMPs across the network that ease this revealing process. The solution of the inverse optimization problem allows revealing each offer price (by rival producers) that is marginal at
least once and thus sets the corresponding LMP. Since the inverse optimization problem is linear and well behaved, it solution is achieved in an efficient and robust manner.

REFERENCES


Carlos Ruiz received the Ingeniero Industrial degree and the Ph.D. degree from the Universidad de Castilla-La Mancha, Ciudad Real, Spain, in 2007 and 2012, respectively.

He is currently a postdoctoral researcher at Universidad Carlos III de Madrid, Madrid, Spain. His research interests include optimization, statistics, MPECs and EPECs, and their applications to the study of energy markets.

Antonio J. Conejo (F’04) received the M.S. degree from the Massachusetts Institute of Technology, Cambridge, in 1987 and the Ph.D. degree from the Royal Institute of Technology, Stockholm, Sweden, in 1990.

He is currently a full Professor at the Universidad de Castilla-La Mancha, Ciudad Real, Spain. His research interests include control, operations, planning and economics of electric energy systems, as well as statistics and optimization theory and its applications.

Dimitris J. Bertsimas (M’11) received the B.S. degree in electrical engineering and computer science at the National Technical University of Athens, Athens, Greece, in 1985, the M.S. degree in operations research at the Massachusetts Institute of Technology (MIT), Cambridge, in 1987, and the Ph.D. degree in applied mathematics and operations research at MIT in 1988.

He is currently the Boeing Professor of Operations Research and the Codirector of the Operations Research Center at MIT and has been with the MIT faculty since 1988. His research interests include optimization, stochastic systems, data mining, and their application. In recent years he has worked in robust optimization, health care, and finance.

Dr. Bertsimas is a member of the National Academy of Engineering, and he has received numerous research awards including the Farkas Prize (2008), the Erlang Prize (1996), the SIAM Prize in Optimization (1996), the Bodnanssaki Prize (1998), and the Presidential Young Investigator Award (1991–1996).